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A guidance law with finite time convergence accounting for autopilot lag ☆

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ABSTRACT

By considering the dynamics of a missile's autopilot as a first-order lag, a guidance law with finite time convergence is designed based on target–missile relative motion equations. It is rigorously proved that states of the guidance system converge to a sliding-mode in finite time and the line-of-sight (LOS) angular rate converges to zero in finite time under the proposed guidance law. Simulation results show that the guidance law is robust against target maneuvers and is able to compensate for the autopilot lag.

1. Introduction

In the past, missiles gained the advantage over intercepted targets in terms of speed, mobility, and agility, and so the proportional navigation guidance law had been widely used [16]. Furthermore, optimal guidance laws were presented to realize an interception with minimum energy or minimum time. For example, minimum time intercept trajectories for air-to-air tactical missiles were proposed in [17] by solving a two-point boundary problem. Recently, to intercept targets with high maneuverability, such as ballistic missiles, robust guidance laws have been extensively studied. Many existing robust guidance laws, such as H_∞ guidance law [18], L_2 gain guidance law [20], Lyapunov-based nonlinear guidance law [8], and first-order sliding-mode guidance laws [19,11] are obtained based on Lyapunov theorems on asymptotic stability or exponential stability. They cannot guarantee a finite time convergence. The theoretical results only indicated that the line-of-sight (LOS) angular rate under the above guidance laws converges to zero or a small neighborhood of zero as time approaches infinity. These theoretical findings are inconsistent with practical observations. In many applications, the time of termination is really quite short. For example, in an endoatmospheric interception where a missile intercepting a ballistic target, sometimes, the time of terminal guidance is only several seconds such that the guidance law is required to ensure finite time convergence of the LOS angular rate.

In recent years, the finite time stability for feedback control systems (i.e., the states of the systems converge to their equilibrium point and then stay there) has become an active research area. Finite time control, which is related to finite time stability, was first proposed in [5] in 1986. It has since generated many research activities in the following two decades, see, for example, [2,9,4,13]. The fundamental tool for analyzing the finite time stability of control systems is the second method of Lyapunov.

Guidance laws with finite time convergence were first proposed based on second-order sliding-mode controls. They are naturally obtained by applying second-order sliding-mode controls to guidance law design [10], but rather complex in structure. A guidance scheme with finite time convergence was proposed in [21] based on Lyapunov scalar differential inequality. Its complexity is just comparable to that of a first-order sliding-mode guidance law. However, in the design of the first-order sliding-mode like guidance law with finite time convergence, the missile's autopilot lag was neglected.

In practical applications, the autopilot lag of a homing missile usually causes significant bad influence on the precision of guidance, especially in the presence of target maneuvers. Thus, it is necessary to consider the autopilot lag in the design of guidance laws. Many integrated designs for missile autopilot and guidance law have been proposed based on optimal control [14,1], Lyapunov stability theorem [12,3], and sliding-mode control [15,6,7]. However, no results have been found about integrating the first-order sliding-mode like guidance law with finite time convergence with the autopilot design.

In this paper, the Lyapunov scalar differential inequality and the non-singular terminal sliding-mode control method [4] are

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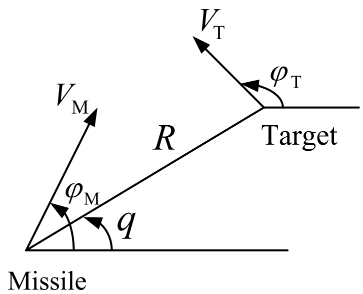

Fig. 1. Planar missile–target engagement.

employed to design a finite time convergent guidance law accounting for the missile's autopilot lag. The guidance law is still like a first-order sliding-mode guidance law in structure and so it is simpler than the integrated design with second-order sliding-mode control. In Section 2, the equations of a planar relative motion are integrated with an equation representing the missile's autopilot lag. Design methods for guidance laws with finite time convergence will be obtained in Section 3. Simulation results are provided in Section 4 to verify the effectiveness of the proposed guidance law. Some concluding remarks are made in Section 5.

2. Formulation of missile–target engagement

In order to simplify the equations of the pursuit situation, it is assumed that the missile and the target are point masses moving in plane. Then, the missile–target engagement model shown in Fig. 1 can be described by the following nonlinear differential equations:

$$\dot{R} = V_T \cos(q - \varphi_T) - V_M \cos(q - \varphi_M) \tag{1}$$

$$R\dot{q} = -V_T \sin(q - \varphi_T) + V_M \sin(q - \varphi_M) \tag{2}$$

where R represents the target–missile relative range; V_T and V_M denote the velocities of the target and missile, respectively; q denotes the LOS angle; and φ_T and φ_M represent the flight path angles of the target and missile, respectively.

Differentiating Eqs. (1) and (2) with respect to time yields

$$\ddot{R} = R\dot{q}^2 + a_{TR} - a_{MR} \tag{3}$$

$$\ddot{q} = -\frac{2\dot{R}}{R}\dot{q} + \frac{1}{R}a_{Tq} - \frac{1}{R}a_{Mq} \tag{4}$$

where, a_{TR} and a_{MR} denote the accelerations of the target and missile along the LOS, respectively; a_{Tq} and a_{Mq} denote the accelerations of the target and missile normal to the LOS, respectively. Usually, in the process of terminal guidance, only the acceleration normal to missile's velocity can be adjusted and so we just discuss the relative motion normal to the LOS. Under an assumption that the rate of relative range $\dot{R}$ is negative, the objective for the design of a guidance law is to find a control variable a_{Mq} such that $\dot{q}$ is nullified.

In addition, we assume that the dynamics of the missile autopilot can be approximately described by the following first-order term:

$$\dot{a}_{Mq} = -\frac{1}{\tau}a_{Mq} + \frac{1}{\tau}u \tag{5}$$

where τ is the time constant of the autopilot, and u is the command to the autopilot.

Let $x_1 = \dot{q}$ and $x_2 = \dot{x}_1$. Substituting them into Eq. (4) gives

$$x_2 = -a_g x_1 - b_g a_{Mq} + b_g a_{Tq} \tag{6}$$

where

$$a_g = \frac{2\dot{R}}{R}, \qquad b_g = \frac{1}{R} \tag{7}$$

It is obvious from Eq. (6) that

$$a_{Tq} = a_{Mq} + \frac{1}{b_g}(a_g x_1 + x_2) \tag{8}$$

Differentiating Eq. (6) with respect to time, it follows from Eqs. (5) and (8) that

$$\dot{x}_2 = A_1 x_1 + A_2 x_2 + bu - b a_{Mq} + f \tag{9}$$

where

$$A_1 = -\dot{a}_g + \frac{\dot{b}_g}{b_g}a_g, \qquad A_2 = -a_g + \frac{\dot{b}_g}{b_g}$$

$$b = -\frac{b_g}{\tau}, \qquad f = b_g \dot{a}_{Tq} \tag{10}$$

Transform Eqs. (6) and (9) into the following state-space form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ A_1 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u - \begin{bmatrix} 0 \\ b a_{Mq} \end{bmatrix} + \begin{bmatrix} 0 \\ f \end{bmatrix} \tag{11}$$

In Eq. (11), f is viewed as a bounded external disturbance, i.e., $\|f\| \leqslant \Delta$, where $\Delta = \text{const.} > 0$.

3. Guidance law with finite-time convergence accounting for autopilot lag

3.1. Finite-time stability of nonlinear systems

In order to design a finite-time convergent guidance law, some results about finite-time stability of nonlinear systems [21] are introduced here.

Definition. Consider a nonlinear system in the form of

$$\dot{x} = f(x, t), \qquad f(0, t) = 0, \qquad x \in R^n \tag{12}$$

where $f : U_0 \times R \to R^n$ is continuous on $U_0 \times R$, and U_0 is an open neighborhood of the origin $x = 0$. The state of the system is said to converge to its local equilibrium $x = 0$ in finite time, if for any given initial time t_0 and initial state $x(t_0) = x_0 \in U$, there exists a settling time $T \geqslant 0$, which is dependent on x_0, such that every solution of the system (12), $x(t) = \upsilon(t; t_0, x_0) \in U/\{0\}$, satisfies

$$\begin{cases} \lim_{t \to T(x_0)} \upsilon(t; t_0, x_0) = 0, & t \in [t_0, T(x_0)) \\ \upsilon(t; t_0, x_0) = 0, & t > T(x_0) \end{cases} \tag{13}$$

Moreover, if the system equilibrium $x = 0$ (local) is Lyapunov stable and is finite-time convergent in a neighborhood of the origin $U \subset U_0$, then the system equilibrium is called finite-time stable. If $U = R^n$, then the origin is a global finite-time stable equilibrium.

The following lemma [21] will prove useful in the design of guidance law.

Lemma 1. *Consider the nonlinear system described by Eq. (12). Suppose that there is a C^1 (continuously differentiable) function $V(x, t)$ defined in a neighborhood $\widehat{U} \subset R^n$ of the origin, and that there are real numbers $\alpha > 0$ and $0 < \lambda < 1$, such that $V(x, t)$ is positive-definite on $\widehat{U}$ and that $\dot{V}(x, t) + \alpha V^{\lambda}(x, t) \leqslant 0$ on $\widehat{U}$. Then, the zero solution of system (12) is finite-time stable.*

Remark 1. Note that if $\widehat{U} = R^n$ and $V(x, t)$ is radially unbounded, then the origin is globally finite-time stable.

3.2. Finite-time convergent guidance law with autopilot lag

For the time-varying guidance system (11), a terminal sliding variable is described by the following equation

$$S = x_1 + \beta x_2^{p/q} \tag{14}$$

where β is a positive constant and p and q are positive odd integers.

We obtain the results presented in the following theorem.

Theorem 1. *For the guidance system (11) with the sliding variable (14), if the guidance law is designed as*

$$u = \frac{-A_1x_1 - A_2x_2 + ba_{Mq} - \frac{1}{\beta}\frac{q}{p}x_2^{2-p/q} - \varepsilon \operatorname{sgn} s}{b} \tag{15}$$

where $\varepsilon = \Delta + \eta$, $\eta > 0$, $\beta > 0$, and $1 < p/q < 2$, then the system states reach the sliding mode $S = 0$ in finite time. Furthermore, the LOS angular rate x_1 and its derivative x_2 converge to zero in finite time.

Proof. At the initial time $t = 0$, the initial values of the system states are denoted as R_0, $\dot{R}_0$, x_{10}, and x_{20}. At time t, they are denoted as $R(t)$, $\dot{R}(t)$, $x_1(t)$, and $x_2(t)$.

Differentiating Eq. (14) with respective to time gives

$$\begin{aligned}\dot{S} &= \dot{x}_1 + \beta\frac{p}{q}x_2^{p/q-1}\dot{x}_2 \\ &= x_2 + \beta\frac{p}{q}x_2^{p/q-1}(A_1x_1 + A_2x_2 + bu - ba_{Mq} + f) \\ &= \beta\frac{p}{q}x_2^{p/q-1}(f - \varepsilon \operatorname{sgn} s)\end{aligned} \tag{16}$$

Define a Lyapunov function

$$V_1 = S^2 \tag{17}$$

The derivative of V_1 along the trajectories of Eq. (16) satisfies

$$\dot{V}_1 = 2\beta\frac{p}{q}x_2^{p/q-1}\big(fS - \varepsilon|S|\big) \leqslant -2\beta\eta\frac{p}{q}x_2^{p/q-1}|S| \tag{18}$$

Since p and q are positive odd integers and $1 < p/q < 2$, there is $x_2^{p/q-1} > 0$ for $x_2 \neq 0$. Let $\rho(x_2) = 2\beta\eta(p/q)x_2^{p/q-1}$, then there are $\rho(x_2) > 0$ and $\dot{V}_1 \leqslant -\rho(x_2)|S|$ when $x_2 \neq 0$. Thus, the system satisfies the Lyapunov stability theory.

Substituting Eq. (15) into Eq. (11) gives

$$\dot{x}_2 = -\frac{1}{\beta}\frac{q}{p}x_2^{2-p/q} + f - (\Delta + \eta)\operatorname{sgn} S \tag{19}$$

When $x_2 = 0$, Eq. (19) can be written as

$$\dot{x}_2 = f - (\Delta + \eta)\operatorname{sgn} S \tag{20}$$

It is clear from Eq. (20) that $\dot{x}_2 \leqslant -\eta$ when $S > 0$ while $\dot{x}_2 \geqslant \eta$ when $S < 0$. In the guidance system, if $S \neq 0$, then according to Eq. (20) it holds that $x_2 \neq 0$ and there exists a small positive constant σ such that $\sigma \leqslant |x_2|$. Thus, the following inequality is satisfied:

$$\rho(x_2) \geqslant 2\beta\eta\frac{p}{q}\sigma^{p/q-1} \tag{21}$$

By Lemma 1 and Eqs. (18) and (21), the system states reach the sliding mode $S = 0$ in finite time and the settling time is given by

$$t_{r1} \leqslant \frac{|S(0)|}{\beta\eta\frac{p}{q}\sigma^{p/q-1}} \tag{22}$$

where $S(0) = x_{10} + \beta x_{20}^{p/q}$.

Let t_{r1} denote the time of the system states reaching the sliding mode $S = 0$. The corresponding system states are represented by $x_1(t_{r1})$ and $x_2(t_{r1})$. In the sliding mode $S = 0$, the following equation is satisfied:

$$x_1 + \beta x_2^{p/q} = 0 \tag{23}$$

Construct a Lyapunov function

$$V_2 = x_1^2 \tag{24}$$

Differentiating V_2 along the trajectory of Eq. (23) results in

$$\dot{V}_2 = -2x_1\frac{1}{\beta^{q/p}}x_1^{q/p} = -\gamma V_2^{\lambda} \tag{25}$$

where $\gamma = 2/\beta^{q/p}$, $\lambda = (q/p + 1)/2$. By Lemma 1, the guidance system states x_1 and x_2 converge to zero in finite time and the settling time is $t_{r2} + t_{r1}$, where t_{r2} is given by

$$t_{r2} = \frac{V_2(t_{r1})^{1-\lambda}}{\gamma(1-\lambda)} = \frac{\beta^{q/p}p}{p-q}\big|x_1(t_{r1})\big|^{1-q/p} \qquad \square \tag{26}$$

It should be noted that the proposed guidance law (15) is nonsingular if $1 < p/q < 2$. In practical applications, the rate of relative range $\dot{R}$ can be approximately viewed as a constant, i.e.,

$$\dot{R} = -c, \qquad \ddot{R} = 0, \qquad c = \text{const.} > 0 \tag{27}$$

Combining Eqs. (7) and (27) leads to

$$a_g = \frac{-2c}{R}, \qquad b_g = \frac{1}{R}, \qquad \dot{a}_g = \frac{-2c^2}{R^2}, \qquad \dot{b}_g = \frac{c}{R^2} \tag{28}$$

It follows from Eqs. (28) and (10) that

$$A_1 = 0, \qquad A_2 = \frac{3c}{R}, \qquad b = -\frac{1}{R\tau} \tag{29}$$

Then, substituting Eq. (29) into the guidance law (15) gives

$$u = 3c\tau x_2 + a_{Mq} + R\tau\frac{1}{\beta}\frac{q}{p}x_2^{2-p/q} + R\tau\varepsilon \operatorname{sgn} S \tag{30}$$

For convenience, in the following discussions the guidance law (30) is called a finite time convergent guidance law with autopilot lag (FTCGAL).

The FTCGAL is a non-smooth controller which guarantees fast convergence and robustness of the guidance system. It involves a signum function, indicating that the control variable sometimes switch. In a practical system, the switching cannot be completely instantaneous. The delay of switching induces the chattering effect. To remove the chattering, the signum function can be smoothened, usually replaced with a saturation function $\operatorname{sat}_\delta(x)$, which is expressed as

$$\operatorname{sat}_\delta(S) = \begin{cases} 1, & S > \delta \\ S/\delta, & |S| \leqslant \delta \\ -1, & S < -\delta \end{cases} \tag{31}$$

where δ is a small positive constant.

3.3. Finite-time convergent guidance law with autopilot lag in three-dimensional model

Consider the spherical LOS coordinates (R, θ, ϕ) with origin fixed at the missile's gravity center. Let (e_R, e_θ, e_ϕ) be the unit vectors along the coordinate axes (see Fig. 2). By virtue of the principles of kinematics, the three relative acceleration components (a_R, a_θ, a_ϕ) can be expressed by the following set of second-order nonlinear differential equations [21]:

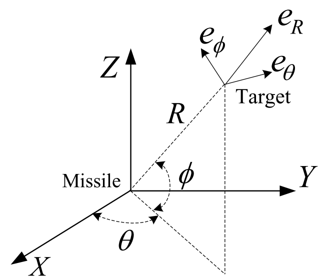

Fig. 2. Three-dimensional interception geometry.

$$\ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi = a_{TR} - a_{MR} \equiv a_R \tag{32a}$$

$$R\ddot{\theta}\cos\phi + 2\dot{R}\dot{\theta}\cos\phi - 2R\dot{\phi}\dot{\theta}\sin\phi = a_{T\theta} - a_{M\theta} \equiv a_\theta \tag{32b}$$

$$R\ddot{\phi} + 2\dot{R}\dot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi = a_{T\phi} - a_{M\phi} \equiv a_\phi \tag{32c}$$

In the process of terminal guidance, only the acceleration normal to missile's velocity can be adjusted and so we just discuss the relative motion normal to the LOS. The purpose of designing a guidance law is to nullify the LOS angular rates $\dot{\theta}$ and $\dot{\phi}$. When ϕ is also a small variable, it gives $\cos\phi \approx 1$, the decoupled three-dimensional LOS angular motion is equivalent to two planar LOS angular motions. The autopilot lag can be usually described by the following first-order differential equations:

$$\dot{a}_{M\theta} = -\frac{1}{\tau}a_{M\theta} + \frac{1}{\tau}u_1 \tag{33a}$$

$$\dot{a}_{M\phi} = -\frac{1}{\tau}a_{M\phi} + \frac{1}{\tau}u_2 \tag{33b}$$

By virtue of the results obtained in Section 3.2, then the two planar finite-time convergent guidance laws with autopilot lag can be designed as

$$\begin{aligned} u_1 &= 3c\tau\ddot{\theta} + a_{M\theta} + R\tau\frac{1}{\beta}\frac{q}{p}\ddot{\theta}^{2-p/q} + R\tau\varepsilon_1\,\mathrm{sat}_\delta(S_1) \\ u_2 &= 3c\tau\ddot{\phi} + a_{M\phi} + R\tau\frac{1}{\beta}\frac{q}{p}\ddot{\phi}^{2-p/q} + R\tau\varepsilon_2\,\mathrm{sat}_\delta(S_2) \end{aligned} \tag{34}$$

where $S_1 = \dot{\theta} + \beta\ddot{\theta}^{p/q}$ and $S_2 = \dot{\phi} + \beta\ddot{\phi}^{p/q}$.

4. Numerical simulations

Define an inertial reference coordinate system which is parallel to the coordinate system, *MXYZ*, as shown in Fig. 2. This system is inertially fixed and is centered at launch site at the instant of the launch. In this system, the X-axis is taken to be in the horizontal plane and in the direction of the launch, the positive Z-axis is in the vertical plane, and the Y-axis is chosen in such a way that the coordinate system forms a right-handed coordinate system. The interceptor's initial position coordinates are $x_{M0} = 0$ m, $y_{M0} = 0$ m, and $z_{M0} = 0$ m. Its initial velocity is $V_{M0} = 1500$ m/s and its initial flight path and heading angles are $\varphi_{M0} = 30°$ and $\psi_{M0} = 0°$ respectively. The target's initial position coordinates are $x_{T0} = 16$ km, $y_{T0} = 10.4$ km, and $z_{T0} = 6$ km. Its initial velocity is $V_{T0} = 1000$ m/s and its initial flight path and heading angles are $\varphi_{T0} = -15°$ and $\psi_{T0} = 145°$, respectively.

In consideration of practical aerodynamic parameters, the missile's autopilot is designed around some typical points on an ideal flight trajectory. On each typical point, a controller is designed for a group of given aerodynamic parameters with classical frequency domain method. In both the elevation loop and the azimuth loop, the missile's control system consists of an inner loop feedback by the missile body's angular rate and an outer loop feedback by the missile's normal acceleration. Although this practical autopilot is a higher order element, in the guidance law design it is approximately thought as a first-order lag. The following simulation results will prove that such an approximation is reasonable. In the terminal guidance process, the fuel of engine has been burnt out and so there is no thrust in the simulation.

Table 1
Summary of miss distances.

Guidance law	Miss distance, m
ASMG	1.05
FTCG	0.82
FTCGAL	0.01

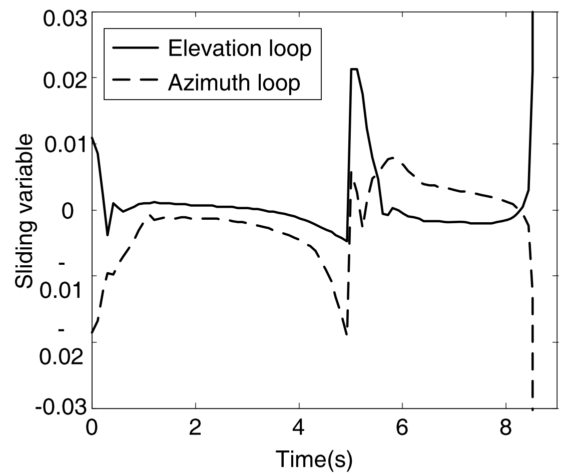

Fig. 3. The sliding variable.

As $t < 5$ s, the target acceleration in the elevation loop is set to be $a_{T\phi} = -20g$ and that in the azimuth loop is set to be $a_{T\theta} = 20g$. As $t \geqslant 5$ s, they are set to be $a_{T\phi} = 20g$ and $a_{T\theta} = -20g$, respectively. The maximum acceleration of the missile is $40g$. The sampling period of the missile's autopilot is 2.5 ms and that of the target seeker is 20 ms.

It was shown that the proportional navigation guidance law gives rise to a large miss distance under the above conditions [21]. To verify the effectiveness of the FTCGAL, the adaptive sliding-mode guidance law (ASMG) [19] and the finite-time convergent guidance law (FTCG) [21] are also simulated under the same conditions. The ASMG and FTCG are expressed as $u_1 = 5|\dot{R}|\dot{\theta} + 200\,\mathrm{sat}_\delta(\dot{\theta})$, $u_2 = 5|\dot{R}|\dot{\phi} + 200\,\mathrm{sat}_\delta(\dot{\phi})$ and $u_1 = 5|\dot{R}|\dot{\theta} + \hat{a}_{T\theta} + 10|\dot{\theta}|^{0.1}\,\mathrm{sgn}\,\dot{\theta}$, $u_2 = 5|\dot{R}|\dot{\phi} + \hat{a}_{T\phi} + 10|\dot{\phi}|^{0.1}\,\mathrm{sgn}\,\dot{\phi}$, respectively, where $\hat{a}_{T\theta}$, $\hat{a}_{T\phi}$ denote an estimate of the target acceleration. In the FTCGAL, the parameters are set to be $\beta = 2$, $p = 7$, $q = 5$, $\varepsilon = 0.02$, $\delta = 0.001$ and $\tau = 0.1$. In both the elevation loop and the azimuth loop, the same guidance laws are employed.

Table 1 summarizes the miss distances resulted from the different guidance laws. The sliding variable is plotted in Fig. 3. Since the simulation results in the elevation loop and those in the azimuth loop are similar. In the following cases, only the results in the elevation loop are plotted. The LOS angular rate, the LOS angular acceleration, the acceleration command, the angle of attack and the elevator deflection angle are plotted in Figs. 4 through 8, respectively.

Fig. 3 indicates that the under the FTCGAL the terminal sliding variable converges to a small neighborhood of sliding-mode in finite time even while encountering strong target maneuvers. Figs. 4 and 5 illustrate that under the FTCGAL the LOS angular rate and the LOS angular acceleration also converge to zero in finite time along with the system states converging to the sliding mode $S = 0$. Fig. 6 illustrates that under the FTCGAL the missile's acceleration command converges to the target acceleration along with

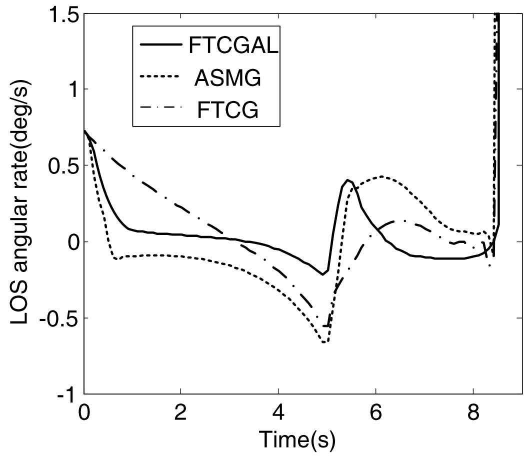

Fig. 4. The LOS angular rate in elevation loop.

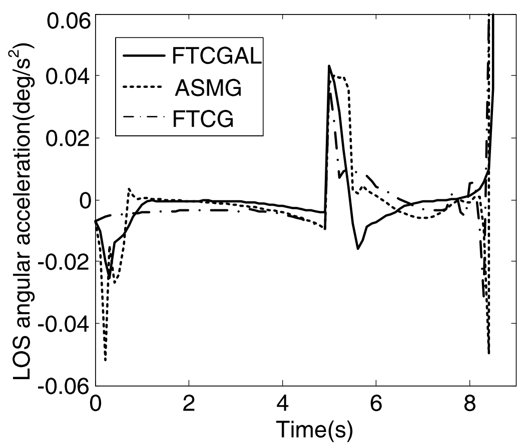

Fig. 5. The LOS angular acceleration in elevation loop.

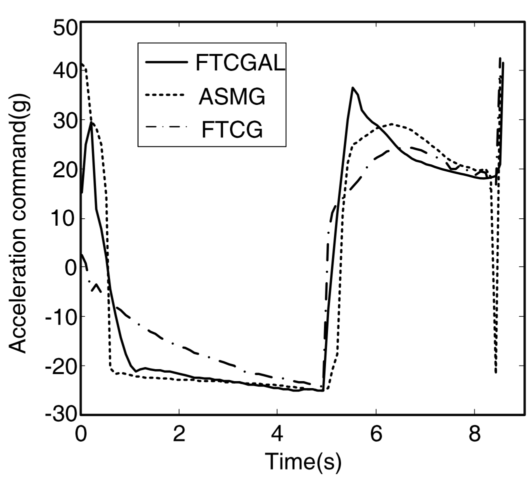

Fig. 6. The acceleration command in elevation loop.

the convergences of the LOS angular rate and angular acceleration. Figs. 7 and 8 show the variations of angle of attack and that of the elevator deflection angle, respectively. At $t = 5$, the target accelerations change their signs abruptly in both the elevation loop and azimuth loop. Due to the lag of autopilot, the change of directions of the missile's normal accelerations cannot be so sharp (see Fig. 6) that the sliding variables and the LOS accelerations are suddenly increased (see Figs. 3 and 5) and the LOS rates are gradually increased (see Fig. 4). However, in the succeeding sev-

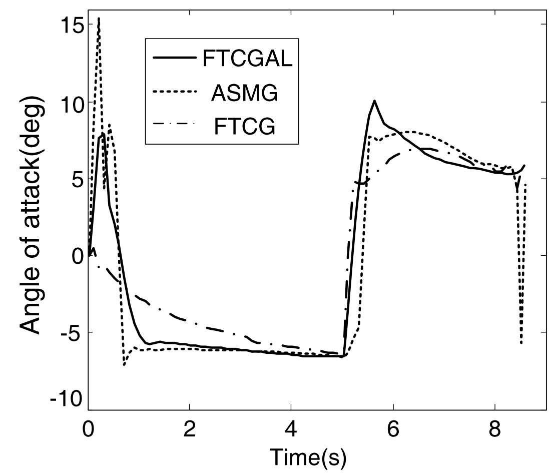

Fig. 7. The angle of attack.

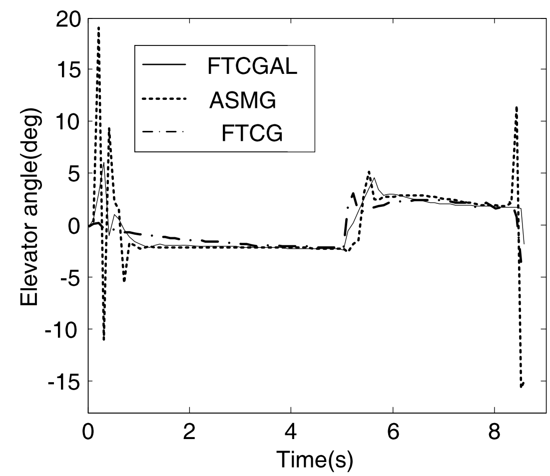

Fig. 8. The elevator deflection angle.

eral seconds, the sliding variables under the FTCGAL converge to the sliding mode $S = 0$ in finite time again (see Fig. 3). In meanwhile, the LOS angular rate and angular acceleration converge to zero in finite time (see Figs. 4 and 5) and the missile accelerations converge to the target accelerations (see Fig. 6). Even though the variables are divergent at the last instant of interception, the guidance accuracy can be ensured. The above results indicate that the FTCGAL has effectively compensated for the influence of autopilot lag.

It was also shown in Figs. 4 through 8 that due to the bad influence of the autopilot lag, under both the ASMG and the FTCG, the guidance system is not suitable for the target maneuver such that the LOS angular rates and angular accelerations diverge earlier and then larger miss distances are yielded (see Table 1). Thus, in the presence of autopilot lag, the ASMG and the FTCG cannot ensure a high guidance precision.

5. Conclusions

The first-order sliding-mode like guidance law with finite-time convergence is extended to the case of considering the dynamics of missile autopilot. By describing the autopilot dynamics as a first-order lag, a finite time convergent guidance law with compensation for the autopilot lag is designed. In practical applications, even though the autopilot is not an ideal first-order lag, the guidance law still performs well. It guides the system states to the sliding-mode in finite time and then guides the LOS angular rate to

zero in finite time in the presence of target maneuvers. The guidance law is able to provide a high guidance precision in short time terminations.

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